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THE VELOCITY PATTERN IN INDENTATION BY A STAMPING TOOL

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The planar deformation is considered when a hard smooth stamping tool enters an elastoplastic medium bounded by a plane. In the limiting state, the tool with a flat base moves downwards with a speed v_0.

1. Continuous Solution for Velocities. Figure 1 shows the network of slip lines corresponding to Prager's solution [3], which is a combination of the solutions due to Prandtl [1] and Hill [2].

In what follows we use not only a Cartesian coordinate system (x, y) but also a curvilinear system (ρ, θ), where

$$x = 1 + \rho \sin \theta, \; y = -\rho \cos \theta.$$

The width of the tool is taken as 2.

The length of the segment A_1B_1 is 2λ. Parameter λ can take any value in the range $0 \leq \lambda \leq 1$ and defines the dimensions of triangle A_1B_1C.

The following is the velocity pattern for the network of Prager slip lines (Fig. 1):

$u = 0,$	$v = -v_0$	in triangle $A_1B_1C,$
$u = v_0,$	$v = -v_0$	in triangle $B_1T_1B,$
$u = v_0/2,$	$v = -v_0/2$	in rectangle $CB_1T_1T,$
$u = \sqrt{2}v_0 \cos \theta,$	$v = \sqrt{2}v_0 \sin \theta$	in segment $T_1B_1D,$
$u = (v_0/\sqrt{2}) \cos \theta,$	$v = (v_0/\sqrt{2}) \sin \theta$	in region $TT_1D_1D,$
$u = v_0,$	$v = v_0$	in triangle $BD_1E_1,$
$u = v_0/2,$	$v = v_0/2$	in region $D_1E_1ED,$

where u and v are the components of the velocity vector along the x and y axes, respectively. The tangential velocity component is discontinuous along the lines CB_1, CTDE, $B_1T_1D_1E_1$. In the limiting cases of $\lambda = 1$ and 0 we obtain the Prandtl and Hill solutions. Other possible velocity solutions have been considered in [4].

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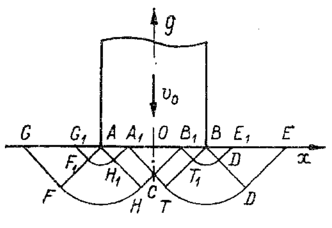

Fig. 1

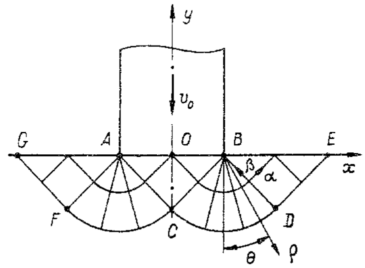

Fig. 2

In the solution considered below, the network of slip lines corresponds to Fig. 2 and coincides with the network of Prandtl slip lines. The velocity pattern takes the form

$$\begin{cases} u = v_0 x, \quad v = -v_0(1+y) & \text{in triangle } ABC, \\ u = -v_0 \cos\theta\,(\rho - \sqrt{2}), & \\ v = -v_0 \sin\theta\,(\rho - \sqrt{2}) & \text{in region } CDB, \\ u = v = -(v_0/2)(x - y - 3) & \text{in triangle } BDE. \end{cases} \tag{1.1}$$

Here the velocity pattern of (1.1) is continuous everywhere in the yield region.

We checked the condition for positive power dissipation:

$$D = \sigma_x e_x + \sigma_y e_y + \tau\gamma,$$

where σ_x, σ_y, τ are the components of the stress tensor and e_x, e_y, γ are the components of the strain-rate tensor,

$$e_x = \partial u/\partial x,\ e_y = \partial v/\partial y,\ \gamma = \partial u/\partial y + \partial v/\partial x.$$

The expression for D may be written as

$$D = \tau_s(-2e_x \sin 2\xi + \gamma \cos 2\xi),$$

where ξ is the inclination of the tangent to the line α (Fig. 2) reckoned in the positive direction from the x axis. In triangle ABC we have $\xi = -\pi/4$, while $\xi = \theta$ in region CBD and $\xi = \pi/4$ in triangle BDE. The expressions for the velocities give us that the components of the strain-rate tensor are

$$\begin{aligned} &e_x = -e_y = v_0,\ \gamma = 0 && \text{in triangle } ABC, \\ &e_x = -e_y = -v_0 \sin 2\theta/\sqrt{2}\rho,\ \gamma = 2v_0 \cos 2\theta/\sqrt{2}\rho && \text{in region } CBD, \\ &e_x = -e_y = v_0/2,\ \gamma = 0 && \text{in triangle } BDE. \end{aligned}$$

The corresponding values of the dissipation power are as follows: in region ABC $2\tau_s v_0$, in region BCD $\sqrt{2}\tau_s v_0/\rho$, and in region BDE $\tau_s v_0$; consequently, the condition for positive dissipation power is obeyed everywhere in the plastic region.

2. Continuous Velocity Pattern as the Limit of a Sequence of Discontinuous Solutions. We consider a segment of unit length OB (Fig. 1) which is divided into n equal parts and put $\lambda_i = i/n$, $0 \le i \le n$; then according to the above for each λ_i there is a discontinuous velocity pattern (u_i^n, v_i^n) corresponding to the Prager solution for $\lambda = \lambda_i$ (Fig. 1). We consider the velocity pattern (u^n, v^n), which is a combination of the solutions (u_i^n, v_i^n) and takes the form

$$u^n = \frac{1}{1+n}\sum_{i=0}^{n} u_i^n, \quad v^n = \frac{1}{1+n}\sum_{i=0}^{n} v_i^n.$$

The velocity patterns (u_i^n, v_i^n) by construction are solutions to the problem. It can be shown that in this case the velocities (u^n, v^n) are also solutions.

We denote by (u_*, v_*) the continuous velocity field defined by (1.1); we now show that in the metric of the space of L_2 functions that are summable in square the velocity field (u_*, v_*) is the limit of the sequence $\{u^n, v^n\}$, i.e.,

$$\lim_{n\to\infty}\int_\Omega (u_* - u^n)^2\, d\Omega = 0, \quad \lim_{n\to\infty}\int_\Omega (v_* - v^n)^2\, d\Omega = 0.$$

We prove the first equality, while the proof for the second is analogous. We have

$$\lim_{n\to\infty}\int_\Omega (u_* - u^n)^2\, d\Omega = \lim_{n\to\infty}\left(\int_\Omega u_*^2 d\Omega - \frac{2}{(1+n)}\sum_{i=0}^{n}\int_\Omega u_* u_i^n d\Omega + \frac{1}{(1+n)^2}\sum_{i,j=0}^{n}\int_\Omega u_i^n u_j^n d\Omega\right). \tag{2.1}$$

We use the form of the functions u_i^n and u_* and also standard formulas from the theory of series to get after integration that

$$\begin{gathered}\int_\Omega u_*^2 d\Omega = \frac{5+\pi}{12}, \quad \frac{1}{(1+n)}\sum_{i=0}^{n}\int_\Omega u_* u_i^n d\Omega = \frac{5+\pi}{12} - \frac{1}{48n}, \\ \frac{1}{(1+n)^2}\sum_{i,j=0}^{n}\int_\Omega u_i^n u_j^n d\Omega = \frac{5+\pi}{12} - \frac{4+\pi-2n}{48n(1+n)}.\end{gathered} \tag{2.2}$$

We substitute (2.2) into (2.1) to get

$$\int_\Omega (u_* - u^n)^2\, d\Omega = \frac{6+\pi}{48n(1+n)},$$

where the right side tends to zero for $n \to \infty$. It has thereby been shown that the continuous solution for the velocities of (1.1) is the limit to the sequence of discontinuous solutions.

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CHANGE IN THE FILTRATION PARAMETERS OF A SATURATED COLLECTOR DUE TO A CONFINED EXPLOSION

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1. It has now been quite definitely established that the permeability of a monolithic rock of granite type increases by up to 4-5 orders of magnitude after a contained explosion by comparison with the initial permeability, which is extremely small (0.01 mD). The permeability of coal after an explosion increases more moderately (by 2 orders), while the initial permeability is of the order of 100 mD [1, 3]. In both cases there is a monotone fall in the permeability to the peripheral initial value away from the explosion cavity. In these media the improvement in the permeability is due to the explosive generation of radial and other crack systems.

On the other hand, a contained explosion in an air-dry porous highly permeable medium leads [4] to a substantial fall in the permeability everywhere around the explosion cavity, in spite of the dilatation. There is marked improvement in the hydraulic permeability due only to passage of individual joints near the explosion cavity. Therefore, the irreversible changes in permeability produced in porous rocks by explosion are due to competing mechanisms of fracturing and pore consolidation. The parameters of the irreversible rock deformation corresponding to appreciable permeability change are the damage [3] (i.e., the jointing) and the extremely small residual strain (0.01%).

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